The University of Calgary
Department of Philosophy

Philosophy 679 Lec 01
TOPICS IN LOGIC:
Topics 01: Gödel's Incompleteness Theorems
Winter 2010 — Richard Zach

Course Outline

Instructor:	**Richard Zach**
Office:	1254 Social Sciences
Office Hours:	MW 2:30–3:30 (tentative) or by appointment
Phone:	(403) 220–3170 email: rzach@ucalgary.ca
Lectures:	**MW 4:00–5:15**
	1253 Social Sciences

Course Description

This course is a complement and in part a continuation of Phil 379 (Logic II). Specifically, we will focus on two famous theorems of symbolic logic due to Kurt Gödel: The Incompleteness Theorems. The first of these states, roughly, that every formal mathematical theory, provided it is sufficiently expressive and free from contradictions, is incomplete in the sense that there are always statements (in fact, true statements) in the language of the theory which the theory can't prove.

In order to prove the Incompleteness Theorem, we'll need to study the expressive power of formal languages and axiomatic theories—this is an important and exciting area in itself. This investigation will lead us naturally to a topic familiar from Logic II, i.e., computability. In Logic III, however, we'll approach computability not via Turing machines, but via the notion of a recursive function. (We will prove, however, that both notions coincide.)

Required Text

Peter Smith, *An Introduction to Gödel's Theorems*, Cambridge University Press, 2007

Requirements and Evaluation

The course requirements are: A "diagnostic" homework assignment (5%), four homework assignments (60%—15% each), and a final project (30%—25% paper, 5% presentation). There will be no exams. You must submit all four assignments and the final project.

The final project will consist in a paper (approx. 2500 words; 3000 words for graduate students) on a topic of your choice related to the material covered in the course. You will give an oral presentation of your paper in the final week of class. Possible projects include: working out a proof only sketched in class in detail, presenting material from a chapter of the book not covered in class, researching a logical topic related to the course, writing a philosophical paper on the importance of Gödel's theorems. The amount and quality of independent research on your part (or lack thereof) will be taken into account in assigning a grade.

Class participation counts for the remaining 5% of your grade. If you are shy and do not want to speak in class, 4 substantive, serious posts over the course of the term on the online discussion board will earn you an A for participation. Only posts made before the last day of class count. If all your posts are made within a 7-day period, you will receive a maximum credit of 2 grade points for them.

On each problem on an assignment and on the paper and presentation parts of the final project you will receive a letter grade reflecting the level of mastery of the material shown by the work you submit. According to the *Calendar*, letter grades are defined as follows:

A Excellent—superior performance, showing comprehensive understanding of subject matter. (Your solution to an assigned problem shows that you understand the problem and how to solve it; the solution is complete and rigorously correct, and is reasonably direct and elegant.)

B Good—clearly above average performance with knowledge of subject matter generally complete. (You understand the problem and give a complete solution, although there may be minor gaps in the proof, or the solution is correct but circuitous.)

C Satisfactory—basic understanding of the subject matter. (You understand what the question is asking but your solution contains significant errors or gaps.)

D Minimal pass—marginal performance. (You show some knowledge of what is asked, but you don't come near a solution.)

F Fail—Unsatisfactory performance. (It is not clear that you understand what the question is asking, or your proposed solution goes completely in the wrong direction.)

The correspondence of letter grades with grade points is defined in the *Calendar* (A = 4, B = 3, C = 2, D = 1, F = 0). "Slash" grades are possible with grade point values 0.5 below the higher grade (e.g., A/B = 3.5).

In computing your final grade, your marks will be converted to grade points and averaged according to the weights given above. The final grade will be the letter grade corresponding to the weighted average of your assignments, exams, and participation plus a margin of 0.2. For the final grade, $+$'s and $-$'s are possible, too; as defined in the *Calendar*, $+/-$ adds/subtracts 0.3 grade points. In other words, a course average of 3.8 or higher receives an A; at least 3.5 and less than 3.8, an A$-$; at least 3.1 and less than 3.5, a B$+$; at least 2.8 and less than 3.1, a B; and so on. (This means that for two A's and an A/B you still get an A overall; for two A's and a B, an A$-$.) There is no D$-$

grade; to earn a D you require a course average of at least 0.8. The A+ grade is reserved for "truly outstanding" performance.

Assignments and Policies

Late work and extensions. Assignments handed in late will be penalized by the equivalent of one grade point per calendar day. If you turn an assignment in late, you must give it to me personally or put it in the department drop-box (it will then be date-stamped by department staff). Note that the drop-boxes are cleared at 4 pm, the department closes at 4:30 pm on weekdays and *is closed Saturdays and Sundays*.

Collaboration. Collaboration on homework assignments is encouraged. However, you must write up your own solutions, and obviously you must not simply copy someone else's solutions. You are also required to list the names of the students with whom you've collaborated on the assignment. **If you collaborate without following these instructions, it constitutes cheating.**

Plagiarism. You might think that it's only plagiarism if you copy a term paper off the Internet. However, you can also plagiarize in a logic course, e.g., by copying a proof verbatim from the textbook (and only making the necessary changes to apply it to the assigned problem.) The point of logic problems which are similar to the proofs in the text is to make you work through those proofs, understand them, and then prove a similar result on the homework. Hence, all homework solutions must be in your own words; copying or paraphrasing closely from the text will be treated as plagiarism and results in a failing grade on the problem; in egregious cases also in a report to the Dean.

Checking your grades and reappraisals of work. University policies for reappraisal of term work and final grades apply (see the *Calendar* section "Reappraisal of Grades and Academic Appeals"). In particular, term work (homework assignments, final paper) will only be reappraised within 15 days of the date you are advised of your marks. Please keep track of your assignments (make sure to pick them up in lecture or in office hours) and your marks (check them on the website) and compare them with the graded work returned to you.

Course Website

A course website on U of C's BlackBoard server has been set up. You should be automatically registered on the first day of class if you're registered in the class. To access the BlackBoard site, you can either go directly to blackboard.ucalgary.ca and log in with your UCIT account name and password, or you can access it through the myUofC portal at my.ucalgary.ca. To sign up for a UCIT account, see the the Student Computer Support website at www.ucalgary.ca/computersupport. ***Please log on at least once by the end of the second week of classes.***

If you are not registered in the course on the first day of class, you will be added to the website within a day of registering.

I will use the email function on BlackBoard to send out important notices. Therefore, ***please make sure your email address in BlackBoard is current.*** For instructions on how to update your email address, see elearn.ucalgary.ca/blackboard/email.

Tentative Syllabus and Due Dates

This is a tentative syllabus to give you a rough idea what parts of the book we will cover when. Due dates are subject to change.

Week 1: Introduction, Review (Jan 11, 13). Chapters 1–3

Learning goals: Understanding enumerability, effective enumerability, decidability; axiomatic theories and their properties.

Week 2: Expressing and Capturing Numerical Properties (Jan 18, 20): Chapters 4–6.

Learning goals: Understanding how properties and relations of numbers can be expressed in the language of arithmetic, and how they can be captured by formal theories. Understanding the connections between expressive power of theories and decidability.

Diagnostic Assignment due.

Week 3: Formal Arithmetics (Jan 25, 27): Chapters 8–10

Learning goals: Acquaintance with formal theories of arithmetic, facility with proving things in these theories. Understanding induction.

Assignment 1 due

Week 4: Primitive Recursive Functions (Feb 1, 3). Chapters 11–12.

Learning goals: Understanding primitive recursion, facility with defining primitive recursive functions.

Week 5: Capturing Primitive Recursive Functions (Feb 8, 10). Chapter 13

Learning goals: Understanding why all p.r. functions are Σ_1, and why all Σ_1 functions can be captured in Q.

Reading week Feb 14–21.

Week 6: Arithmetization of Syntax (Feb 22, 24): Chapter 15.

Learning goals: Understanding Gödel numbering and why syntactic properties are primitive recursive.

Assignment 2 due

Week 7: Incompleteness (Mar 1, 3). Chapter 16, 17

Learning goals: Understanding and proving Gödel's First Incompleteness Theorem.

Week 8: Diagonalization (Mar 8, 10): Chapters 19–21

Learning goals: Generalizing the proof of the first incompleteness theorem; understanding and applying the diagonalization lemma to get Tarski's Theorem about the undefinability of truth.

Week 9: Provability Predicates and the Second Incompleteness Theorem (Mar 15, 17): Chapters 24–27.

Learning goals: Understanding formalized consistency statements and provability conditions. The second incompleteness theorem. Reflection principles.

Assignment 3 due

Week 10: Recursive Functions (Mar 22, 24): Chapters 29, 30.

Learning goals: Understanding μ-recursive functions.

Week 11: Recursive Functions and Turing Machines (Mar 29, 31): Chapter 31–34.

Learning goals: Understanding why μ-recursive functions are Turing computable and vice versa. The Church-Turing Thesis.

Project draft due

Week 12: Advanced Topics (April 5, 7).

Assignment 4 due

Week 13: Student Presentations (April 12, 14).

Final project due

INTELLECTUAL HONESTY

Intellectual honesty is the cornerstone of the development and acquisition of knowledge and requires that the contribution of others be acknowledged. As a result, cheating or plagiarism on any assignment or examination is regarded as an extremely serious academic offence, the penalty for which may be an F on the assignment and possibly also an F in the course, academic probation, or requirement to withdraw. The University Calendar states that plagiarism exists when:

- the work submitted or presented was done, in whole or in part, by an individual other than the one submitting or presenting the work (this includes having another impersonate the student or otherwise substituting the work of another for one's own in an examination or test);
- parts of the work are taken from another source without reference to the original author;
- the whole work (e.g., an essay) is copied from another source; and/or
- a student submits or presents work in one course which has also been submitted in another course (although it may be completely original with that student) without the knowledge of or prior agreement of the instructor involved.

While it is recognized that scholarly work often involves reference to the ideas, data and conclusions of other scholars, intellectual honesty requires that such references be explicitly and clearly noted. Plagiarism is an extremely serious offence. Plagiarism occurs not only when direct quotations are taken from a source without specific acknowledgement, but also when original ideas or data from the source are not acknowledged. A bibliography is insufficient to establish which portions of the student's work are taken from external sources; footnotes or other recognized forms of citation must be used for this purpose.

CHEATING

Cheating is an extremely serious academic offence. Cheating at tests or examinations includes but is not limited to dishonest or attempted dishonest conduct such as speaking to other candidates or communicating with them under any circumstances whatsoever; bringing into the examination room any textbook, notebook, memorandum, other written material or mechanical or electronic device not authorized by the examiner; writing an examination or part of it, or consulting any person or materials outside the confines of the examination room without permission to do so, or leaving answer papers exposed to view, or persistent attempts to read other students' examination papers.

ACADEMIC ACCOMMODATION

It is the student's responsibility to request academic accommodation. If you are a student with a documented disability who may require academic accommodation and have not registered with the Disability Resource Centre, please contact their office at 220-8237. Students who have not registered with the Disability Resource Centre are not eligible for formal academic accommodation. You are also required to discuss your needs with your instructor no later than fourteen (14) days after the start of this course.

STUDENTS' UNION REPRESENTATIVE

The Humanities Representative is Britney Luimes, MSC 251, humanitiesrep@su.ucalgary.ca or 220-3913.