



**UNIVERSITY OF CALGARY**  
FACULTY OF ARTS  
Department of Philosophy

## **PHIL 677 Lec 01**

### **Metalogic**

**Winter Term 2020**

**MWF 13:00–13:50, Science Theatres 126**

#### **Course Outline**

**Instructor:** [Richard Zach](#) (he/him)  
**Email:** [rzach@ucalgary.ca](mailto:rzach@ucalgary.ca)  
**Office:** Social Sciences 1230  
**Office Hours:** MW 12:00–12:50, or by appointment  
**Phone:** (403) 220–3170

Email is preferred over phone. However please keep the following in mind:

1. Please ensure that “Phil 677” or some other clearly identifying term occurs in the subject line. Otherwise there is a strong possibility that your message will be deleted unread as spam.
2. If you email to make an appointment please indicate the times when you are available.
3. Please make sure your first and last names are clearly included in the body of any email message.
4. We will not respond to email after 7pm or on weekends.

#### **Course information**

Formal logic has many applications both within philosophy and outside (especially in mathematics, computer science, and linguistics). This second course will introduce you to the concepts, results, and methods of formal logic necessary to understand and appreciate these applications as well as the limitations of formal logic. It will be mathematical in that you will be required to master abstract formal concepts and to prove theorems *about* logic

(not just *in* logic the way you did in Phil 279); but it does not presuppose any (advanced) knowledge of mathematics. We will start from the basics.

We will begin by studying some basic formal concepts: sets, relations, and functions, and the sizes of infinite sets. We will then consider the language, semantics, and proof theory of first-order logic (FOL), and ways in which we can use first-order logic to formalize facts and reasoning about some domains of interest to philosophers, computer scientists, and logicians.

In the second part of the course, we will begin to investigate the meta-theory of first-order logic. We will concentrate on a few central results: the completeness theorem, which relates the proof theory and semantics of first-order logic, and the compactness theorem and Löwenheim-Skolem theorems, which concern the existence and size of first-order structures.

In the third part of the course, we will discuss a particular way of making precise what it means for a function to be computable. To this end, we will discuss a “model of computation”: Turing machines. We will show that there are problems which are *undecidable* in the sense that there is no Turing machine which, in a finite amount of time, provides a definite yes-or-no answer. The first example of an undecidable problem is the *halting problem*, i.e., the problem of deciding, given the description of a Turing machine, whether it halts on a given input. We will also show that the *decision problem*—i.e., the problem of deciding, given a sentence of first-order logic, whether it is valid—is undecidable.

If there is time, we will cover some advanced topics at the end of the semester, such as second-order logic or solvable cases of the decision problem.

### **Prerequisites**

There are no prerequisites for this course.

### **Course objectives**

By the end of the course, you should be able to ...

1. Understand, construct, and formulate simple mathematical proofs in which you apply definitions, identify hypotheses, and correctly and appropriately use informal patterns of mathematical reasoning.
2. Understand and apply the methods of definition by induction and proof by induction, both for the natural numbers and for inductively defined sets such as the set of formulas of first-order logic.

3. Understand and use the vernacular of set theory (sets, relations, functions) to describe and explain the metalogical properties of the model and proof theory of first-order logic as well as of Turing machines and computable functions, and to prove intermediate facts about infinite sets and their sizes using, e.g., the diagonal method of Cantor's theorem.
4. Understand the formal syntax, model theory, and proof theory of first-order logic, to explain the definitions, the properties, and the relationships between logical notions (free and bound variables, sentences, satisfaction, consequence and satisfiability, inference rule, derivation, provability and consistency), and to prove intermediate facts about them (such as the soundness of the proof system).
5. Understand the completeness theorem of first-order logic, to explain the overall structure as well as the individual steps of the proof, to explain and prove from it corollaries such as the compactness and Löwenheim-Skolem theorems, and to apply these to properties of theories and the size of models thereof.
6. Understand the concept of Turing machines, how they can be used to define computable functions, to construct simple Turing machines, to formulate and prove the undecidability of the halting problem, and to formulate and explain the decision problem.

### Required readings

The textbook

*Sets, Logic, Computation: An Open Introduction to Metalogic* (Fall 2019 edition)

is required and will be made available electronically via D2L.

### Course website

The course has a [D2L/Brightspace](#) site. You will find course readings, assignments, quizzes and tests, grades, a discussion board there. Any updates including revisions to office hours, PASS sessions, course schedule, and deadlines will be posted there. **To make sure you don't miss a deadline or an important update, please review your [notification settings](#).**

## **Assessment Components**

**There will be no registrar-scheduled final exam.**

*Basic problems.* There will be 10 short-answer basic problems. They will usually be due on Fridays, to be submitted at the beginning of class or to the PHIL 379 drop-box on the 12th floor of the Social Sciences building by 13:00. The first problem will be due January 24. Basic problems are graded according to the EMRN rubric below.

*Challenge problems.* There will be 4 additional short answer challenge problems. Challenge problems are graded according to the EMRN rubric below.

### **How your work is evaluated**

Basic and Challenge Problems are graded using the four-level EMRN rubric, illustrated below. Marks of **E** (exemplary) and **M** (meets expectations) are “passing”. Marks of **R** (needs revision) and **N** (not assessable) are not passing.

Please note that none of the work in the class is assessed using points. **Your progress toward a grade in the course is determined simply by the quantity of passing marks you earn on various assignments and how many E marks you receive on basic and challenge problems.** This is a “competency based” approach to grading that gives you full control over how you earn your grade and provides transparency as to what you have mastered and what you still need to work on.

To receive a pass grade in the class, students must earn a passing grade on at least 9 basic problems and at least 2 challenge problems, with an E grade on at least 4 problems total, at least 1 of which must be a challenge problem.

### **EMRN rubric**

- If your work demonstrates thorough understanding of the concepts and meets the expectations outlined in the assignment...
  - and the work is complete and well documented, it earns an **Exemplary (E)** mark.  
(The work meets or exceeds the expectations of the assignment. Communication is clear and complete. Mastery of concepts is evident. There are no nontrivial errors. This work could be used as a classroom example.)

- otherwise, it earns a **Meets Expectations (M)** mark.  
(Understanding of the concepts is evident through correct work and clear explanations. Some revision or expansion is needed, but no significant gaps or errors are present. No additional instruction on the concepts is needed.)
- if not, but ...
  - there is evidence of partial understanding, it earns a **Revisions Needed (R)** mark.  
(Partial understanding of the concepts is evident, but significant gaps remain. Needs further work, more review, and/or improved explanations.)
  - otherwise, it earns a **Not Assessable (N)** mark.  
(Not enough evidence is present in the work to determine whether there is understanding of the concepts. The work is fragmentary, contains significant errors or omissions, or there are too many issues to justify correcting each one.)

### **Online quizzes**

On the D2L site you will find weekly pre-class quizzes and weekly tests that are completed by the students in 379. You are welcome to take these quizzes and tests to gauge your own mastery of the concepts; however, they will not count towards a passing grade in the class.

*Revision.* You have the opportunity to revise almost any item of work in the class if you want to raise your grade on it. Specifically:

- You can resubmit any basic or challenge problem on which you received a grade of at least R once, up to seven calendar days after the graded problem was returned.
- You can use a token (see below) to revise any basic or challenge problem on which you received a grade of N.

*Tokens.* Tokens are a kind of currency for this class. Each student has three (3) tokens to spend to bend the rules of the class in various ways. You can use a token to do the following.

- Revising a problem on which you received an N grade (see above).
- Submitting a problem by the deadline for revisions if you did not turn it in.
- Purchase a no-questions-asked extension of 48 hours on any basic or challenge problem.

### **Course policies**

*Late policy.* Assignments will not normally be accepted after the deadlines unless special permission has been given by the instructor. Failure to submit an assignment or test on time will normally result in a mark of zero. Students who cannot submit an assignment or a test due to medical reasons or other reasonable grounds should contact the instructor as soon as possible.

*Plagiarism.* You might think that it's only plagiarism if you copy a term paper off the internet. However, you can also plagiarize in a logic course, e.g., by copying a proof verbatim from the textbook or the internet (and only making the necessary changes to apply it to the assigned problem.) The point of logic problems which are similar to the proofs in the text is to make you work through those proofs, understand them, and then prove a similar result on the problem sets. Hence, all solutions must be in your own words; copying or paraphrasing closely from the text or elsewhere constitutes plagiarism, which must be reported to the Dean's office by university policy. It may result in a failing grade or worse penalties.

*Checking your grades and reappraisals of work.* University policies for reappraisal of term work and final grades apply (see the *Calendar* section "[Reappraisal of Grades and Non-Disciplinary Academic Appeals](#)"). In particular, term work will only be reappraised within 10 calendar days of the date you are advised of your marks. Please keep track of your assignments (make sure to pick them up in lecture or in office hours) and your marks (check them on D2L) and compare them with the graded work returned to you.

### **Schedule of topics and due dates**

Due dates are tentative and subject to revision.

Week	Topics and readings (chapters)	Q	T	BP	CP
1	(Jan 13) Sets and proofs about sets (1, A)	1			
2	(Jan 20) Relations; proofs by induction (2, B)	2		1	
3	(Jan 27) Function and enumerations (3)	3	1	2	
4	(Feb 3) Countable and uncountable sets (4)	4	2	3	
5	(Feb 10) Syntax and semantics of FOL (5)	5	3	4	1
6	(Feb 24) Theories (6)	6	4	5	
7	(Mar 2) Natural deduction (7, 9)	7	5		2
8	(Mar 9) Completeness (10)	8		6	
9	(Mar 16) Consequences of completeness (10)		6	7	3
10	(Mar 23) Turing machines (12)	9	7	8	
11	(Mar 30) The halting problem, reductions (13)	10	8	9	
12	(Apr 6) The decision problem (13)		9	10	4
13	(Apr 13) Additional topics		10		

### **Important departmental, faculty, and university information**

*Academic accommodations.* It is the student's responsibility to request academic accommodations according to the University policies and procedures. The student accommodation policy can be found at: [ucalgary.ca/policies/files/policies/student-accommodation-policy.pdf](http://ucalgary.ca/policies/files/policies/student-accommodation-policy.pdf)

Students seeking an accommodation because of a disability or medical condition should communicate this need to Student Accessibility Services in accordance with the Procedure for Accommodations for Students with Disabilities: [ucalgary.ca/policies/files/policies/procedure-for-accommodations-for-students-with-disabilities.pdf](http://ucalgary.ca/policies/files/policies/procedure-for-accommodations-for-students-with-disabilities.pdf)

Students needing an accommodation in relation to their coursework or to fulfil requirements for a graduate degree, based on a protected ground other than disability, should communicate this need, preferably in writing, to their instructor.

*Absence or missed course assessments.* Students who are absent from class assessments (tests, participation activities, or other assignments) should inform their instructors as soon as possible. Instructors may request that evidence in the form of documentation be provided. If the reason provided for the absence is acceptable, instructors may decide that any arrangements made can take forms other than make-up tests or assignments. For example, the weight of a missed grade may be added to another assignment or test. For information on possible forms of documentation, including statutory declarations,

please see: [ucalgary.ca/pubs/calendar/current/m-1.html](http://ucalgary.ca/pubs/calendar/current/m-1.html)

*Student support and resources.* Full details and information about the following resources can be found at [ucalgary.ca/registrar/registration/course-outlines](http://ucalgary.ca/registrar/registration/course-outlines)

- Wellness and Mental Health Resources
- Student Success Centre
- Student Ombuds Office
- Student Union (SU) Information
- Graduate Students' Association (GSA) Information
- Emergency Evacuation/Assembly Points
- Safewalk

*Academic Advising.* If you are a student in the Faculty of Arts, you can speak to an academic advisor in the Arts Students' Centre about course planning, course selection, registration, program progression and more. Visit the Faculty of Arts website at [arts.ucalgary.ca/advising](http://arts.ucalgary.ca/advising) for contact details and information regarding common academic concerns.

For questions specific to the philosophy program, please visit [phil.ucalgary.ca](http://phil.ucalgary.ca). Further academic guidance is available by contacting David Dick (Undergraduate Program Director, [dgdick@ucalgary.ca](mailto:dgdick@ucalgary.ca)) or Jeremy Fantl (Honours Advisor, [jfantl@ucalgary.ca](mailto:jfantl@ucalgary.ca)). If you have questions regarding registration, please email Rebecca Lesser (Undergraduate Program Administrator, [phildept@ucalgary.ca](mailto:phildept@ucalgary.ca)).

*Writing assessment and support.* The assessment of all written assignments—and, to a lesser extent, written exam responses—is based in part on writing skills. This includes correctness (grammar, punctuation, sentence structure, etc.), as well as general clarity and organization. Research papers must include a thorough and accurate citation of sources. Students are also encouraged to use Writing Support Services for assistance (one-on-one appointments, drop-in support and writing workshops). For more information, and other services offered by the Student Success Centre, please visit [ucalgary.ca/ssc](http://ucalgary.ca/ssc).

*Online resources and electronic devices.* Important information and communication about this course may be posted on D2L (Desire2Learn), UCalgary's

online learning management system. Visit [ucalgary.service-now.com/it](https://ucalgary.service-now.com/it) for how-to information and technical assistance.

The use of laptop and mobile devices is acceptable when used in a manner appropriate to the course and classroom activities. The use of devices must be exclusively for instructional purposes, and without disruption to the instructor or fellow students. In particular, students are to refrain from accessing websites that may be distracting for fellow learners (e.g. personal emails, Facebook, YouTube), watching movies, or playing games. Devices should be set to silent mode during lectures. Audio or video recording of lectures is not permitted without the written permission of the instructor. Students are responsible for being aware of the University's Internet and email use policy, which can be found at: [ucalgary.ca/policies/files/policies/electronic-communications-policy.pdf](https://ucalgary.ca/policies/files/policies/electronic-communications-policy.pdf) Students violating this policy are subject to discipline under the University of Calgary's Non-Academic Misconduct policy.

*Academic misconduct/honesty.* Cheating or plagiarism on any assignment or examination is as an extremely serious academic offense, the penalty for which will be an F on the assignment or an F in the course, and possibly a disciplinary sanction such as probation, suspension, or expulsion. For information on academic misconduct and its consequences, please see the University of Calgary Calendar at: [ucalgary.ca/pubs/calendar/current/k.html](https://ucalgary.ca/pubs/calendar/current/k.html)

Intellectual honesty requires that your work include adequate referencing to sources. Plagiarism occurs when you do not acknowledge or correctly reference your sources. If you have questions about referencing, please consult your instructor.

*University policies.* The Instructor Intellectual Property Policy is available at: [ucalgary.ca/policies/files/policies/Intellectual%20Property%20Policy.pdf](https://ucalgary.ca/policies/files/policies/Intellectual%20Property%20Policy.pdf) Course materials created by professor(s) (including course outlines, presentations, assignments, and exams) remain the intellectual property of the professor(s). These materials may *not* be reproduced, redistributed or copied without the explicit consent of the professor. The posting of course materials to third party websites such as note-sharing sites without permission is prohibited. Sharing of extracts of these course materials with other students enrolled in the course at the same time may be allowed under fair dealing.

The University of Calgary is under the jurisdiction of the provincial Freedom of Information and Protection of Privacy (FOIP) Act, as outlined at [ucalgary.ca/legalservices/foip](https://ucalgary.ca/legalservices/foip). The instructor (or TA) must return graded as-

signments *directly* to the student UNLESS written permission to do otherwise has been provided.

All students are required to read the University of Calgary policy on Acceptable Use of Material Protected by Copyright ([ucalgary.ca/policies/files/policies/acceptable-use-of-material-protected-by-copyright.pdf](http://ucalgary.ca/policies/files/policies/acceptable-use-of-material-protected-by-copyright.pdf)) and requirements of the copyright act ([laws-lois.justice.gc.ca/eng/acts/C-42/index.html](http://laws-lois.justice.gc.ca/eng/acts/C-42/index.html)).